Probability

24

CHAPTER

Favourites to seize the Olympic flame

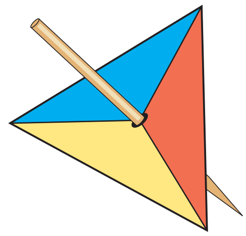
As the day for decision approaches it seems **unlikely** that London will win the battle to host the 2012 Olympic Games. The **probability** that Paris will win this race has always been high. It is felt that Madrid, Moscow and New York have little **chance** of success as the final presentations are made.

London defy all the odds

London won with their bid to host the 2012 Summer Olympic Games. Yesterday's vote saw **likely** winners Paris stumble at the final hurdle. A spokesperson said 'Everyone thought that Paris was **certain** to win the vote but I always felt that we had a greater than **even chance** of success.'

24.1 Writing probabilities as numbers

The diagram shows a three-sided spinner.



The spinner can land on red or blue or yellow. If it is equally likely to land on each of the three colours the spinner is said to be **fair**.

This spinner, which is fair, is spun once. This is called a single **event**.

The colour it lands on is called the **outcome**.

The outcome can be red or blue or yellow. There are three possible outcomes and each possible outcome is equally likely.

The **probability** of an outcome to an event is a measure of how likely it is that the outcome will happen.

The probability that the spinner will land on blue $= \dfrac{\text{1 successful outcome}}{\text{3 possible outcomes}} = \dfrac{1}{3}$

Similarly the probability that the spinner will land on red $= \dfrac{1}{3}$

and the probability that the spinner will land on yellow $= \dfrac{1}{3}$

When all the possible outcomes are equally likely to happen

$$\textbf{probability} = \frac{\textbf{number of successful outcomes}}{\textbf{total number of possible outcomes}}$$

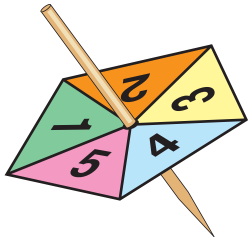
Probability can be written as a fraction or a decimal.

For an event:

- the probability of an outcome which is **certain** to happen is 1 For example the probability that the spinner will land on red or blue or yellow is 1 since the spinner is certain to land on one of these three colours
- the probability of an outcome which is **impossible** is 0 For example the probability that the spinner will land on green is 0 since green is not a colour on the spinner
- all other probabilities lie between 0 and 1

Example 1

A fair five-sided spinner is numbered 1 to 5
Jane spins the spinner once.



a Find the probability that the spinner will land on the number 4

b Find the probability that the spinner will land on an even number.

Solution 1

a The possible outcomes are the numbers 1, 2, 3, 4 and 5 So the total number of possible outcomes is 5 and they are all equally likely.
The 1 successful outcome is the number 4

$$\text{Probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

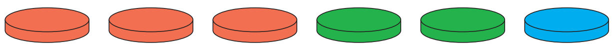
Probability that the spinner will land on the number 4 $= \frac{1}{5}$ or 0.2

b 2 and 4 are the even numbers on the spinner.
The number of successful outcomes is **2**
The total number of possible outcomes is **5**
So the probability that the spinner will land on an even number = $\frac{2}{5}$ or 0.4

In the following example the term **at random** is used. This means that each possible outcome is equally likely.

Example 2

Six coloured counters are in a bag.
3 counters are red, 2 counters are green and 1 counter is blue.



One counter is taken at random from the bag.

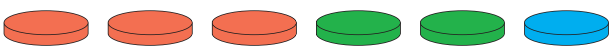
a Write down the colour of the counter which is
  i most likely to be taken **ii** least likely to be taken

b Find the probability that the counter taken will be
  i red **ii** green **iii** blue

Solution 2

a **i** Red is the most likely colour to be taken since the number of red counters is greater than the number of counters of any other colour.
  ii Blue is least likely to be taken since the number of blue counters is less than the number of counters of any other colour.

b There are 6 counters so there are 6 possible outcomes.



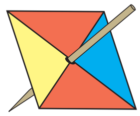
Outcome 1 2 3 4 5 6

  i Number of successful outcomes = 3 (3 red counters)
  Probability that the counter taken will be red $= \frac{3}{6} = \frac{1}{2}$
  ii Number of successful outcomes = 2 (2 green counters)
  Probability that the counter taken will be green $= \frac{2}{6} = \frac{1}{3}$
  iii Number of successful outcomes = 1 (1 blue counter)
  Probability that the counter taken will be blue $= \frac{1}{6}$

Final answers for probabilities written as fractions should be given in their simplest form.

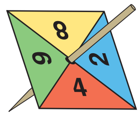
Exercise 24A

1 Nicky spins the spinner. The spinner is fair. Write down the probability that the spinner will land on a side coloured



a blue **b** red **c** green

2 John spins the fair spinner. Write down the probability that the spinner will land on

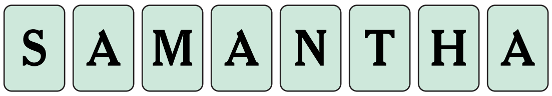


a 2 **b** a number greater than 5

c an even number **d** a number greater than 10

3 Samantha Smith has 8 cards which spell 'Samantha'.

She puts the cards in a bag and chooses one of the cards at random.
Find the probability that she will choose a card showing a



a letter S **b** letter A **c** letter which is also in her surname 'SMITH'

4 Ben has 15 ties in a drawer. 7 of the ties are plain, 3 of the ties are striped and the rest are patterned. Ben chooses a tie at random from the drawer. What is the probability that he chooses a tie which is

a plain **b** striped **c** patterned?

5 Peter has a bag of 8 coins. In the bag he has one 10p coin, five 20p coins and the rest are 50p coins. Peter chooses one coin at random. What is the probability that Peter will choose a

a 10p coin **b** 20p coin **c** 50p coin

d £1 coin **e** coin worth more than 5p?

6 Rob has a drawer of 20 socks. 4 of the socks are blue, 6 of the socks are brown and the rest of the socks are black. Rob chooses a sock at random from the drawer. Find the probability that he chooses

a a blue sock **b** a brown sock **c** a black sock **d** a white sock

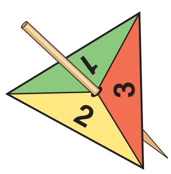
7 Verity has a box of pens. Half of the pens are blue, 11 of the pens are green, 10 of the pens are red and the remaining 4 pens are black. Verity chooses a pen at random from the box.
Find the probability that she chooses

a a blue pen **b** a green pen **c** a red pen **d** a black pen

24.2 Sample space diagrams

A **sample space** is all the possible outcomes of one or more events.

A **sample space diagram** is a diagram which shows the sample space.



For the three-sided spinner, the sample space when the spinner is spun once is
1 2 3

Example 3

The three-sided spinner is spun and a coin is tossed at the same time.
Write down the sample space of all possible outcomes.

Solution 3

There are 6 possible outcomes. For example the spinner landing on 1 and the coin showing heads is written as (1, head). The sample space is

(1, head) (1, tail) (2, head) (2, tail) (3, head) (3, tail)

Example 4

Two fair dice are thrown.

a Write down the sample space showing all the possible outcomes.

b Find the probability that the numbers on the two dice will be
i both the same **ii** both even numbers **iii** both less than 3

Solution 4

a

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

The total number of possible outcomes is 36

b **i** (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) and (6, 6) are the successful outcomes with both numbers the same.
Probability that the numbers on the two dice will be both the same $= \frac{6}{36} = \frac{1}{6}$

ii (2, 2) (2, 4) (2, 6) (4, 2) (4, 4) (4, 6) (6, 2) (6, 4) (6, 6) are the successful outcomes with both numbers even.
Probability that the numbers on the two dice will both be even numbers $= \frac{9}{36} = \frac{1}{4}$

iii (1, 1) (2, 1) (1, 2) (2, 2) are the successful outcomes with both numbers less than 3
Probability that the numbers on the two dice will both be less than 3 is $\frac{4}{36} = \frac{1}{9}$

Exercise 24B

In each of the questions in this exercise give all probabilities as fractions in their simplest forms.

1 Two coins are spun at the same time.

a Write down a sample space to show all possible outcomes.

b Find the probability that both coins will come down heads.

c Find the probability that one coin will come down heads and the other coin will come down tails.

2 A bag contains 1 blue brick, 1 yellow brick, 1 green brick and 1 red brick.
A brick is taken at random from the bag and its colour noted.
The brick is then replaced in the bag.
A brick is again taken at random from the bag and its colour noted.

a Write down a sample space to show all the possible outcomes.

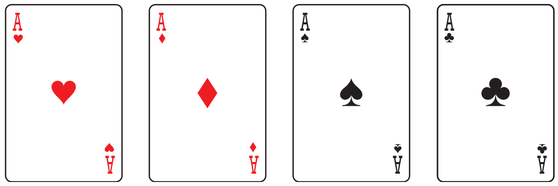
b Find the probability that
i the two bricks will be the same colour
ii one brick will be red and the other brick will be green

3 Two fair dice are thrown. The sample space is shown in Example 4
The numbers on the two dice are added together.

a Find the probability that the sum of the numbers on the two dice will be
  i greater than 10 **ii** less than 6 **iii** a square number.

b **i** Which sum of the numbers on the two dice is most likely to occur?
  ii Find the probability of this sum.

4 Daniel has four cards, the ace of hearts, the ace of diamonds, the ace of spades and the ace of clubs.
Daniel also has a fair dice.
He rolls the dice and takes a card at random.



a Write down the sample space showing all possible outcomes.
One possible outcome, ace of Diamonds and 4 has been done for you, (D, 4).

b Find the probability that a red ace will be taken.

c Find the probability that he will take the ace of spades and roll an even number on the dice.

5 Three fair coins are spun.

a Draw a sample space showing all eight possible outcomes.

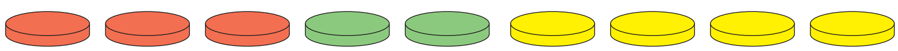
b Find the probability that the three coins will show the same.

c Find the probability that the coins will show two heads and a tail.

d Write down the total number of possible outcomes when
  i four coins are spun **ii** five coins are spun

24.3 Mutually exclusive outcomes and the probability that the outcome of an event will not happen

Nine coloured counters are in a bag.



3 counters are red, 2 counters are green and 4 counters are yellow.
One counter is chosen at random from the bag.
The probability that the counter will be red, P(red) $= \frac{3}{9}$

Notation: 'P(red)' means the probability of red.

P(green) $= \frac{2}{9}$ P(yellow) $= \frac{4}{9}$

Mutually exclusive outcomes are outcomes which cannot happen at the same time.

For example when one counter is chosen at random from the bag the outcome 'red' cannot happen at the same time as the outcome 'green' or the outcome 'yellow'. So the three outcomes are mutually exclusive.

P(red) + P(green) + P(yellow) $= \frac{3}{9} + \frac{2}{9} + \frac{4}{9} = \frac{9}{9} = 1$

The sum of the probabilities of all the possible mutually exclusive outcomes of an event is 1

There are 9 possible outcomes, 2 of which are green.

The probability that the counter will be green is $\frac{2}{9}$

Out of the 9 possible outcomes $9 - 2 = 7$ outcomes are NOT green.

The probability that the counter will NOT be green is $1 - \frac{2}{9} = \frac{7}{9}$

If the probability of an outcome of an event happening is p then the probability of it *NOT* happening is $1 - p$

If the counter is not green it must be either red or yellow.

So, P(not green) = P(either red or yellow)

The probability that the counter will be either red or yellow is $\frac{7}{9}$

$\frac{7}{9} = \frac{3}{9} + \frac{4}{9} =$ P(red) + P(yellow).

So P(either red or yellow) = P(red) + P(yellow)

In general when two outcomes A and B, of an event are mutually exclusive

P(A or B) = P(A) + P(B)

This can be used as a quicker way of solving some problems.

Example 5

David buys one newspaper each day. He buys the *Times* or the *Telegraph* or the *Independent*. The probability that he will buy the *Times* is 0.6 The probability that he will buy the *Telegraph* is 0.25

a Work out the probability that David will buy the *Independent*.

b Work out the probability that David will buy either the *Times* or the *Telegraph*.

Solution 5

a P(*Times*) = 0.6
P(*Telegraph*) = 0.25
P(*Independent*) = ?

P(*Times*) means the probability that David will buy the *Times*.

As David buys only one newspaper each day, the three outcomes are mutually exclusive.

P(*Independent*) + 0.6 + 0.25 = 1
P(*Independent*) + 0.85 = 1
P(*Independent*) = 1 − 0.85

The probability that David will buy the *Independent* = 0.15

b P(*Times* or *Telegraph*) = P(*Times*) + P(*Telegraph*)
= 0.6 + 0.25 = 0.85

The probability that David will buy either the *Times* or the *Telegraph* = 0.85

Example 6

The probability that Julie will pass her driving test next week is 0.6
Work out the probability that Julie will *not* pass her driving test next week.

Solution 6

The probability that Julie will *not* pass = 1 − 0.6 = 0.4

Exercise 24C

1 Nosheen travels from home to school. She travels by bus or by car or by tram. The probability that she travels by bus is 0.4 The probability that she travels by car is 0.5

a Work out the probability that she travels by tram.

b Work out the probability that she travels by car or by bus.

2 Roger's train can be on time or late or early. The probability that his train will be on time is 0.15 The probability that his train will be early is 0.6

a Work out the probability that Roger's train will be late.

b Work out the probability that Roger's train will be either on time or early.

3 The probability that Lisa will pass her Maths exam is 0.8
Work out the probability that Lisa will *not* pass her Maths exam.

4 A company makes batteries. A battery is chosen at random. The probability that the battery will not be faulty is 0.97
Work out the probability that the battery will be faulty.

5 Four athletes Aaron, Ben, Carl and Des take part in a race.
The table shows the probabilities that Aaron or Ben or Carl will win the race.

Aaron	Ben	Carl	Des
0.2	0.14	0.3	

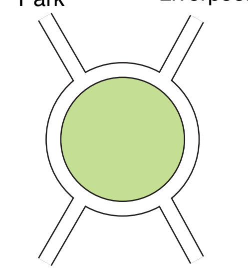
a Work out the probability that Aaron will not win the race.
b Work out the probability that Ben will not win the race.
c Work out the probability that Des will win the race.
d Work out the probability that either Aaron or Ben will win the race.
e Work out the probability that either Aaron or Carl or Des will win the race.

6 The table shows the probabilities of a dice landing on each of the numbers 1 to 6 when thrown.
The dice is thrown once.

Number	Probability
1	0.2
2	0.15
3	0.25
4	0.18
5	0.05
6	0.17

a Work out the probability that the dice will land on either 1 or 3
b Work out the probability that the dice will land on either 2 or 4
c Work out the probability that the dice will land on
 i an even number
 ii an odd number

7 A roundabout has four roads leading from it. Michael is driving round the roundabout. The roads lead to Liverpool or Trafford Park or Eccles or Bolton. The table shows the probabilities that Michael will take the road to Liverpool or Trafford Park or Bolton.



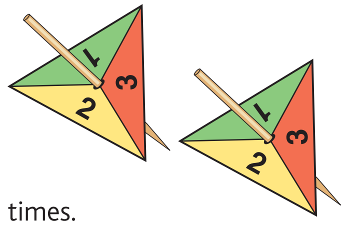
Liverpool	Trafford Park	Eccles	Bolton
0.49	0.18	x	0.23

a Work out the probability that Michael will not take the road to Liverpool.
b Work out the value of x.
c Work out the probability that Michael will take either the road to Trafford Park or the road to Bolton.

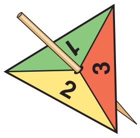
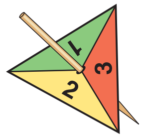
8 Sam has red, white, yellow and green coloured T-shirts only. She chooses a T-shirt at random.
The probabilities that Sam will choose a red T-shirt or a white T-shirt are given in the table.
Sam is twice as likely to choose a green T-shirt as she is to choose a yellow T-shirt.
Work out the value of x.

Red	White	Yellow	Green
0.5	0.14	x	$2x$

24.4 Estimating probability from relative frequency



The diagram shows two three-sided spinners.
One spinner is fair and one is biased.
A spinner is **biased** if it is not equally likely to land on each of the numbers. This can be tested by experiment.
If a spinner is spun 300 times it is fair if it lands on each of the numbers approximately 100 times.
John spins one spinner 300 times and Mary spins the other spinner 300 times.

John's spinner			Mary's spinner		
					
300 spins			300 spins		
1	2	3	1	2	3
97	104	99	147	96	57
FAIR spinner			**BIASED** spinner		

John's spinner is fair because it lands on each of the three numbers approximately the same number of times.
Mary's spinner is biased because it is more likely to land on the number **1**
It is least likely to land on the number **3**
To estimate the probability that Mary's spinner will land on each number, the **relative frequency** of each number is found using

$$\textbf{relative frequency} = \frac{\textbf{number of times the spinner lands on the number}}{\textbf{total number of spins}}$$

Relative frequency that Mary's spinner will land on the number **1** $= \frac{147}{300} = 0.49$
Relative frequency that Mary's spinner will land on the number **2** $= \frac{96}{300} = 0.32$
Relative frequency that Mary's spinner will land on the number **3** $= \frac{57}{300} = 0.19$
An estimate of the probability that the spinner will land on the number **1** is 0.49
An estimate of the probability that the spinner will land on the number **2** is 0.32
An estimate of the probability that the spinner will land on the number **3** is 0.19
If Mary spins the spinner a further 500 times, an estimate for the number of times the spinner lands on the number 2 is $0.32 \times 500 = 160$
In general

if the probability that an experiment will be successful is p and the experiment is carried out N times, then an estimate for the number of successful experiments is $p \times N$.

Example 7

In a statistical experiment Brendan throws a dice 600 times.
The table shows the results.

Number on dice	1	2	3	4	5	6
Frequency	48	120	180	96	54	102

Brendan throws the dice again.
a Find an estimate of the probability that he will throw a 2
b Find an estimate of the probability that he will throw an even number.
Zoe now throws the same dice 200 times.
c Find an estimate of the number of times she will throw a 6

Solution 7

a Estimate of probability of a 2 is $\frac{120}{600} = 0.2$

b The number of times an even number is thrown $= 120 + 96 + 102 = 318$

Estimate of probability of an even number $= \frac{318}{600} = 0.53$

c Estimate of probability of a 6 is $\frac{102}{600} = 0.17$

An estimate for the number of times Zoe will throw a 6 in 200 throws $= 0.17 \times 200 = 34$

Exercise 24D

1 A coin is biased. The coin is tossed 200 times.
It lands on heads 140 times and it lands on tails 60 times.

a Write down the relative frequency of the coin landing on tails.

b The coin is to be tossed again. Estimate the probability that the coin will land on
i tails **ii** heads.

2 A bag contains a red counter, a blue counter, a white counter and a green counter. Asif takes a counter at random. He does this 400 times.

Red	Blue	White	Green
81	110	136	73

The table shows the number of times each of the coloured counters is taken.

a Write down the relative frequency of Asif taking the *red* counter.

b Write down the relative frequency of Asif taking the *white* counter. Asif takes a counter one more time. Estimate the probability that this counter will be

c **i** blue **ii** green.

3 Tyler carries out a survey about the words in a newspaper. He chooses an article at random.
He counts the number of letters in each of the first 150 words of the article.
The table shows Tyler's results.

Number of letters in a word	1	2	3	4	5	6	7	8	9	10
Frequency	7	14	42	31	21	13	10	6	4	2

A word is chosen at random from the 150 words.

a Write down the most likely number of letters in the word.

b Estimate the probability that the word will have
i 1 letter **ii** 7 letters **iii** more than 5 letters.

c The whole article has 1000 words. Estimate the total number of 3-letter words in this article.

4 A bag contains 10 coloured bricks. Each brick is white or red or blue. Alan chooses a brick at random from the 10 bricks in the bag and then replaces it in the bag.

White	Red	Blue
290	50	160

He does this 500 times. The table shows the numbers of each coloured brick chosen.

a Estimate the number of red bricks in the bag.

b Estimate the number of white bricks in the bag.

5 The probability that someone will pass their driving test at the first attempt is 0.45
On a particular day, 1000 people will take the test for the first time.
Work out an estimate for the number of these 1000 people who will pass.

6 Gwen has a biased coin. When she spins the coin the probability that it will come down tails is $\frac{3}{5}$
Work out an estimate for the number of tails she gets when she spins her coin 400 times.

7 The probability that a biased dice will land on a 1 is 0.09 Andy is going to roll the dice 300 times.
Work out an estimate for the number of times the dice will land on a 1

24.5 Independent events

In Example 3 a fair three-sided spinner is spun and a fair coin is tossed at the same time. The outcomes from spinning the spinner do not affect the outcomes from tossing the coin. The outcomes from tossing the coin do not affect the outcomes from spinning the spinner.

These are **independent events** since an outcome of one event does not affect the outcome of the other event.

What is the probability that the spinner will land on 3 and the coin will land on tails? This is written as P(3, tail).

$P(3) = \frac{1}{3} \quad P(\text{tail}) = \frac{1}{2}$

To work out P(3, tail) the sample space could be used. The sample space is

(1, head) (1, tail) (2, head) (2, tail) (3, head) (3, tail)

$P(3, \text{tail}) = \frac{1}{6}$ since this is 1 out of 6 possible outcomes.

But $P(3, \text{tail}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

so $P(3, \text{tail}) = P(3) \times P(\text{tail})$

In general when the outcomes, A and B, of two events are independent

$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$

Example 8

A bag contains 4 green counters and 5 red counters. A counter is chosen at random and then replaced in the bag. A second counter is then chosen at random. Work out the probability that for the counters chosen

a both will be green **b** both will be red **c** one will be green and one will be red

Solution 8

a $P(G) = \frac{4}{9}$

Find the probability that a green counter will be chosen.

$P(\text{G and G}) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

The choosing of the two counters are two independent events so use P(A and B) = P(A) × P(B)

The probability that both counters chosen will be green $= \frac{16}{81}$

b $P(R) = \frac{5}{9}$

Find the probability of choosing a red counter.

$P(\text{R and R}) = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

Use P(A and B) = P(A) × P(B)

The probability that both counters chosen will be red $= \frac{25}{81}$

c P(one G and one R) = P(first G and second R or first R and second G)

Use P(A or B) = P(A) + P(B)

= P(first G and second R) + P(first R and second G)

Hint: A *or* B → *Add* probabilities
A *and* B → *Multiply* probabilities

$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9}$

Use P(A and B) = P(A) × P(B)

$= \frac{20}{81} + \frac{20}{81}$

$P(\text{one G and one R}) = \frac{40}{81}$

The probability that one of the counters chosen will be green and one will be red $= \frac{40}{81}$

Note: (G, G) (R, R) (G, R) (R, G) is the full sample space so answers to parts **a**, **b** and **c** must add up to 1

Exercise 24E

1 A biased coin and a biased dice are thrown. The probability that the coin will land on heads is 0.6
The probability that the dice will land on an even number is 0.7

a Write down the probability that the coin will not land on heads.

b Find the probability that the coin will land on heads and that the dice will land on an even number.

c Find the probability that the coin will *not* land on heads and that the dice will *not* land on an even number.

2 A basket of fruit contains 3 apples and 4 oranges. A piece of fruit is picked at random and then returned to the basket. A second piece of fruit is then picked at random.
Work out the probability that for the fruit picked

a both will be apples

b both will be oranges

c one will be an apple and one will be an orange.

3 Eric and Frank each try to hit the bulls-eye.
They each have one attempt.
The events are independent.

The probability that Eric will hit the bulls-eye is $\frac{2}{3}$

The probability that Frank will hit the bulls-eye is $\frac{3}{4}$

a Find the probability that both Eric and Frank will hit the bulls-eye.

b Find the probability that just one of them will hit the bulls-eye.

c Find the probability that neither of them will hit the bulls-eye.

4 When Edna rings the health centre the probability that the phone is engaged is 0.35
Edna needs to ring the health centre at 9 am on both Monday and Tuesday.
Find the probability that at 9 am the phone will

a be engaged on both Monday and Tuesday

b not be engaged on Monday but will be engaged on Tuesday

c be engaged on at least one day

5 Mrs Rashid buys a car.

Fault	Engine	Brakes
Probability	0.05	0.1

The table shows the probability of different mechanical faults.
Find the probability that the car will have

a a faulty engine and faulty brakes

b no faults

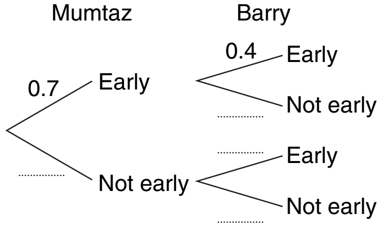
c **exactly** one fault

24.6 Probability tree diagrams

It is often helpful to use **probability tree diagrams** to solve probability problems.
A probability tree diagram shows all of the possible outcomes of more than one event by following all of the possible paths along the branches of the tree.

Example 9

Mumtaz and Barry are going for an interview.
The probability that Mumtaz will arrive early is 0.7
The probability that Barry will arrive early is 0.4
The two events are independent.



a Complete the probability tree diagram.

b Work out the probability that Mumtaz and Barry will both arrive early.

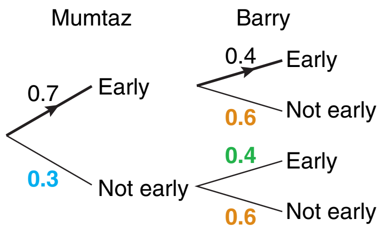
c Work out the probability that just one person will arrive early.

Solution 9

a Mumtaz not early: $1 - 0.7 = 0.3$ — Must be either early or not early.

Barry not early: $1 - 0.4 = 0.6$ — Must be either early or not early.

Barry early: 0.4 — The two events are independent.



P(Early, Early) — This probability is found in part **b**
P(Early, Not early)
P(Not early, Early) — These probabilities are added in part **c**
P(Not early, Not early)

b P(Mumtaz early and Barry early)
$= \text{P(Early, Early)} = 0.7 \times 0.4 = 0.28$

Use P(A and B) = P(A) × P(B) When moving along a path multiply the probabilities on each of the branches.

c P(just one person early)
= P(Mumtaz early and Barry not early OR Mumtaz not early and Barry early)
= P(Early, Not early) + P(Not early, Early)
$= (0.7 \times 0.6) + (0.3 \times 0.4)$
$= 0.42 + 0.12 = 0.54$
P(just one person early) = 0.54

Possible ways for just one person to be early.
Use P(A or B) = P(A) + P(B)

Use P(A and B) = P(A) × P(B)

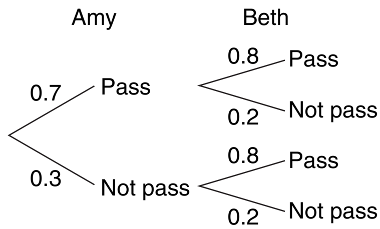
Exercise 24F

1 Amy and Beth are going to take a driving test tomorrow.
The probability Amy will pass the test is 0.7
The probability Beth will pass the test is 0.8
The probability tree diagram shows this information.
Use the probability tree diagram to work out the probability that

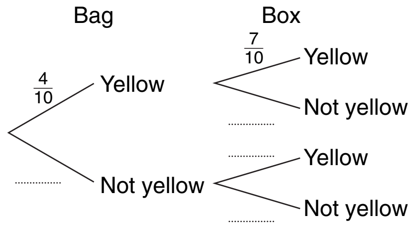
a both women will pass the test

b only Amy will pass the test

c neither woman will pass the test.



2 A bag contains 10 coloured counters, 4 of which are yellow.
A box also contains 10 coloured counters, 7 of which are yellow.
One counter is chosen at random from the bag and one counter is chosen at random from the box.



a Copy and complete the probability tree diagram.

b Find the probability that

i both counters will be yellow

ii the counter from the bag will be yellow and the counter from the box will not be yellow

iii at least one counter will be yellow.

3 The probability that a biased coin will show heads when thrown is 0.4
Tina throws the coin twice and records her results.

a Draw a probability tree diagram.

b Use your diagram to work out the probability that the coin will show

i heads on both throws **ii** heads on *exactly* one throw.

4 The probability that Jason will receive one DVD for his birthday is $\frac{4}{5}$
The probability that he will receive one DVD for Christmas is $\frac{3}{8}$
These two events are independent.
Find the probability that Jason will receive at least one DVD.

5 Stuart and Chris each try to score a goal in a penalty shoot-out.
They each have one attempt.
The probability that Stuart will score a goal is 0.75
The probability that Chris will score a goal is 0.64

a Work out the probability that both Stuart and Chris will score a goal.

b Work out the probability that exactly one of them will fail to score a goal.

24.7 Conditional probability

The probability of an outcome of an event that is dependent on the outcome of a previous event is called **conditional probability**. For example when choosing two pieces of fruit without replacing the first one, the choice of the second piece of fruit is dependent on the choice of the first.

Example 10

A bowl of fruit contains 3 apples and 4 bananas. A piece of fruit is chosen at random and eaten. A second piece of fruit is then chosen at random. Work out the probability that for the two pieces of fruit chosen

a both will be apples

b the first will be an apple and the second will be a banana

c at least one apple will be chosen.

Solution 10

a 1st choice: $P(A) = \frac{3}{7}$

Find the probability that the *first* piece of fruit will be an apple. There is a total of 7 pieces, 3 of which are apples.

2nd choice: $P(A) = \frac{2}{6}$

1st choice was apple so there are now only 6 pieces of fruit, 2 of which are apples. Find the probability that the *second* piece of fruit will be an apple.

$P(A \text{ and } A) = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

Probability both will be apples $= \frac{1}{7}$

Multiply the probabilities.

b 1st choice: $P(A) = \frac{3}{7}$

Find the probability that the *first* piece of fruit will be an apple. There is a total of 7 pieces, 3 of which are apples.

2nd choice: $P(B) = \frac{4}{6}$

1st choice was apple so there are now only 6 pieces of fruit, 4 of which are bananas. Find the probability that the *second* piece of fruit will be a banana.

$P(A \text{ and } B) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$

Multiply the probabilities.

Probability the first will be an apple and the second will be a banana $= \frac{2}{7}$

c P(At least one apple) $= 1 - P(B, B)$

(A, A) (A, B) (B, A) (B, B) are all the possible outcomes so P(At *least* one apple) + P(B, B) = 1

1st choice: $P(B) = \frac{4}{7}$

Find the probability that the *first* piece of fruit will be a banana. There is a total of 7 pieces, 4 of which are bananas.

2nd choice: $P(B) = \frac{3}{6}$

1st choice was banana so there are now only 6 pieces of fruit, 3 of which are bananas. Find the probability that the *second* piece of fruit will be a banana.

$P(B, B) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

Multiply the probabilities.

P(At least one apple) $= 1 - \frac{2}{7} = \frac{5}{7}$

Example 11

There are 4 red crayons, 3 blue crayons and 1 green crayon in a box.
A crayon is taken at random and *not* replaced. A second crayon is then taken at random.

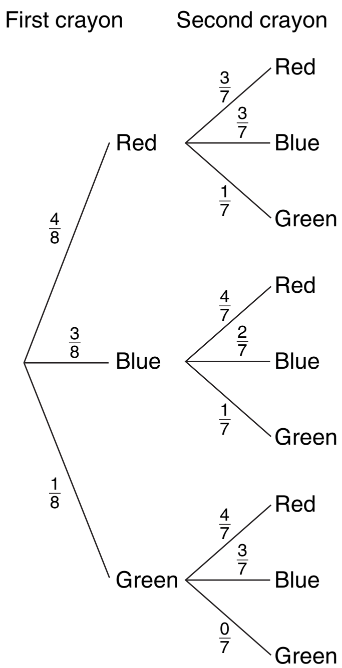
a Draw and complete a probability tree diagram.

b Find the probability that both crayons taken will be **i** blue **ii** the same colour.

c Find the probability that *exactly* one of the crayons will be red.

Solution 11

a First crayon: total of 8 crayons out of which 4 are red, 3 are blue, 1 is green.
Second crayon: total of 7 crayons (since 1st not replaced)
When 1st crayon is red, 3 red, 3 blue, 1 green remain. When 1st crayon is blue, 4 red, 2 blue, 1 green remain. When 1st crayon is green, 4 red, 3 blue, 0 green remain



$P(R, R) = \frac{4}{8} \times \frac{3}{7}$

$P(R, B) = \frac{4}{8} \times \frac{3}{7}$

$P(R, G) = \frac{4}{8} \times \frac{1}{7}$

$P(B, R) = \frac{3}{8} \times \frac{4}{7}$

$P(B, B) = \frac{3}{8} \times \frac{2}{7}$

$P(B, G) = \frac{3}{8} \times \frac{1}{7}$

$P(G, R) = \frac{1}{8} \times \frac{4}{7}$

$P(G, B) = \frac{1}{8} \times \frac{3}{7}$

$P(G, G) = \frac{1}{8} \times \frac{0}{7}$

These results are used later in the question.

b **i** $P(B \text{ and } B) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

Probability that both colours will be blue $= \frac{3}{28}$

Follow the branches 'blue to blue' and multiply the probabilities.

ii Probability that colours will be the same

$= P(R \text{ and } R \text{ or } B \text{ and } B \text{ or } G \text{ and } G)$

$= P(R, R) + P(B, B) + P(G, G)$

$= \frac{4}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{2}{7} + \frac{1}{8} \times \frac{0}{7}$

$= \frac{12}{56} + \frac{6}{56} + \frac{0}{56} = \frac{18}{56}$

Probability that colours will be the same $= \frac{9}{28}$

Colours are either both red or both blue or both green.

c Probability of *exactly* one red

$= P(R, B) + P(R, G) + P(B, R) + P(G, R)$

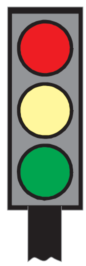
$= \frac{4}{8} \times \frac{3}{7} + \frac{4}{8} \times \frac{1}{7} + \frac{3}{8} \times \frac{4}{7} + \frac{1}{8} \times \frac{4}{7}$

$= \frac{12}{56} + \frac{4}{56} + \frac{12}{56} + \frac{4}{56} = \frac{32}{56}$

Probability of *exactly* one red $= \frac{4}{7}$

Follow the pairs of branches which have just one red.

Example 12



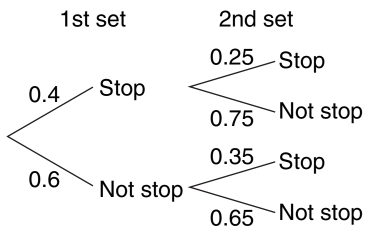
When driving to the shops Rose passes through two sets of traffic lights.
If she stops at the first set of lights the probability that she stops at the second set of lights is 0.25
If she does not stop at the first set of lights the probability that she stops at the second set is 0.35
The probability that Rose stops at the first set of lights is 0.4

a Draw and complete a probability tree diagram.

b Find the probability that when Rose next drives to the shops she will not stop at the second set of traffic lights.

Solution 12

a



b P(Not stopping at 2nd set) $= P(S, NS \text{ or } NS, NS)$

$= P(S, NS) + P(NS, NS)$

$= 0.4 \times 0.75 + 0.6 \times 0.65$

$= 0.3 + 0.38$

Probability that Rose will not stop at the 2nd set $= 0.68$

Exercise 24G

1 A box of chocolates contains 10 milk chocolates and 12 plain chocolates. Two chocolates are taken at random without replacement.
Work out the probability that

a both chocolates will be milk chocolates

b at least one chocolate will be a milk chocolate

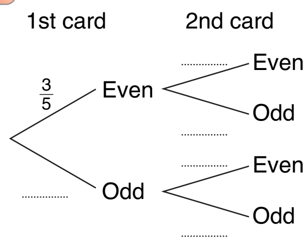
2 Anil has 13 coins in his pocket, 6 pound coins, 3 twenty-pence coins and 4 two-pence coins. He picks two coins at random from his pocket. Work out the probability that the two coins each have the same value.

3 Mandy has these five cards
Each card has a number on it.
She chooses two cards at random without replacement and records the number on each card.

a Copy and complete the probability tree diagram.

b Find the probability that both numbers are even.

c Find the probability that the sum of the two numbers is an even number.



4 Michael returns to school tomorrow.
If it is raining the probability that Michael walks to school is 0.3
If it is not raining the probability that Michael walks to school is 0.8
The probability that it will rain tomorrow is 0.1

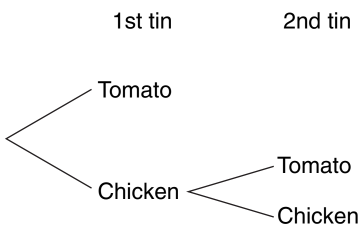
a Draw a probability tree diagram.

b Find the probability that Michael will walk to school tomorrow.

5 A box contains 3 tins of soup. 2 of the tins are chicken soup and 1 is tomato soup.
Betty wants tomato soup. She picks a tin at random from the box. If it is not tomato she gives the tin to her son and then picks another tin at random from the box.

a Copy and complete the probability tree diagram.

b Find the probability that Betty does not pick the tin of tomato soup.



6 The probability that a biased dice when thrown will land on 6 is $\frac{1}{4}$ In a game Patrick throws the biased dice until it lands on 6 Patrick wins the game if he takes no more than three throws.

a Find the probability that Patrick throws a 6 with his second throw of the dice.

b Find the probability that Patrick wins the game.

Chapter summary

You should now know:

★ that **probability** is a measure of how likely the outcome of an event is to happen

★ that probabilities are written as fractions or decimals between 0 and 1

★ that an outcome which is **impossible** has a probability of 0

★ that an outcome which is **certain** to happen has a probability of 1

★ for an event, outcomes which are equally likely have equal probabilities

★ that when calculating probabilities you can use

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

when all outcomes of an event are equally likely to happen
For example the probability of throwing a six on a normal fair dice is $\frac{1}{6}$

- that a **sample space** is all the possible outcomes of one or more events and a **sample space diagram** is a diagram which shows the sample space
- how to list all outcomes in an ordered way using sample space diagrams. For example when two coins are tossed the outcomes are

  (H, H) (T, H)
  (H, T) (T, T)
- that for an event **mutually exclusive outcomes** are outcomes which cannot happen at the same time
- that the sum of the probabilities of all the possible mutually exclusive outcomes is 1
- that if the probability of something happening is p, then the probability of it *NOT* happening is $1 - p$
- that when two outcomes, A and B, of an event are mutually exclusive

  $$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$
- that from a statistical experiment for each outcome

  $$\text{relative frequency} = \frac{\text{number of times the outcome happens}}{\text{total number of trials of the event}}$$
- that relative frequencies give good estimates to probabilities when the number of trials is large
- that if the probability that an experiment will be successful is p and the experiment is carried out a number of times, then an estimate for the number of successful experiments is $p \times$ number of experiments

  For example if the probability that a biased coin will come down heads is 0.7 and the coin is spun 200 times, then an estimate for the number of times it will come down heads is $0.7 \times 200 = 140$
- that for **independent events** an outcome from one event does not affect the outcome of the other event
- that when the outcomes, A and B, of two events are independent

  $$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$$
- that a **probability tree diagram** shows all of the possible outcomes of more than one event by following all of the possible paths along the branches of the tree. When moving along a path multiply the probabilities on each of the branches
- that **conditional probability** is the probability of an outcome of an event that is dependent on the outcome of a previous event. For example choosing two pieces of fruit without replacing the first one where the choice of the second piece of fruit is dependent on the choice of the first.

Chapter 24 review questions

1 Shreena has a bag of 20 sweets.
10 of the sweets are red.
3 of the sweets are black.
The rest of the sweets are white.
Shreena chooses one sweet at random.
What is the probability that Shreena will choose a

a red sweet **b** white sweet? (1385 June 1999)

2 80 students each study one of three languages.
The two-way table shows some information about these students.

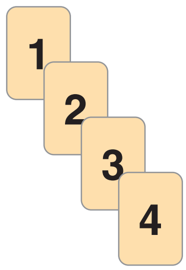
	French	German	Spanish	Total
Female	15			39
Male		17		41
Total	31	28		80

a Copy and complete the two-way table.

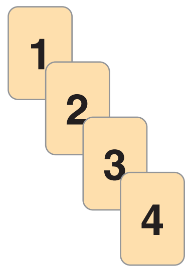
One of these students is to be picked at random.

b Write down the probability that the student picked studies French. (1387 June 2005)

3 Here are two sets of cards.
Each card has a number on it as shown.
A card is selected at random from set A and a card is selected at random from set B.
The difference between the number on the card selected from set A and the number on the card selected from set B is worked out.



Set A



Set B

a Copy and complete the table started below to show all the possible differences.

		Set A			
		1	**2**	**3**	**4**
Set B	**1**	0		2	
	2		0		
	3		1		
	4				

b Find the probability that the difference will be zero.
c Find the probability that the difference will *not* be 2

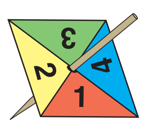
4 There are 20 coins in a bag.
7 of the coins are pound coins.
Gordon is going to take a coin at random from the bag.

a Write down the probability that he will take a pound coin.

b Find the probability that he will take a coin which is *NOT* a pound coin.

5 Mr Brown chooses one book from the library each week.
He chooses a crime novel or a horror story or a non-fiction book.
The probability that he chooses a horror story is 0.4
The probability that he chooses a non-fiction book is 0.15
Work out the probability that Mr Brown chooses a crime novel. (1387 June 2005)

6 Here is a 4-sided spinner. The sides of the spinner are labelled 1, 2, 3 and 4
The spinner is biased.
The probability that the spinner will land on each of the numbers 2 and 3 is given in the table.
The probability that the spinner will land on 1 is equal to the probability that it will land on 4



Number	1	2	3	4
Probability	x	0.3	0.2	x

a Work out the value of x.

Sarah is going to spin the spinner 200 times.

b Work out an estimate for the number of times it will land on 2 (1387 June 2005)

7 Meg has a biased coin.
When she spins the coin the probability that it will come down heads is 0.4
Meg is going to spin the coin 350 times.
Work out an estimate for the number of times it will come down heads.

8 A dice has one red face and the other faces coloured white.
The dice is biased.
Sophie rolled the dice 200 times.
The dice landed on the red face 46 times.
The dice landed on a white face the other times.
Sophie rolls the dice again.

a Estimate the probability that the dice will land on a white face.

Each face of a different dice is either rectangular or hexagonal.
When this dice is rolled the probability that it will land on a rectangular face is 0.85
Billy rolls this dice 1000 times.

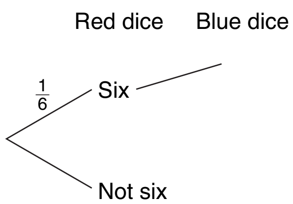
b Estimate the number of times it will land on a rectangular face.

9 Julie does a statistical experiment.
She throws a dice 600 times.
She scores six 200 times.

a Is the dice fair? Explain your answer.

Julie then throws a fair red dice once and a fair blue dice once.

b Copy and complete the probability tree diagram to show the outcomes.
Label clearly the branches of the probability tree diagram.
The probability tree diagram has been started.



c **i** Julie throws a fair red dice once and a fair blue dice once.
Calculate the probability that Julie gets a six on both the red dice and the blue dice.

ii Calculate the probability that Julie gets at least one six.

(1387 June 2003)

10 Lauren and Yasmina each try to score a goal.
They each have one attempt.
The probability that Lauren will score a goal is 0.85
The probability that Yasmina will score a goal is 0.6

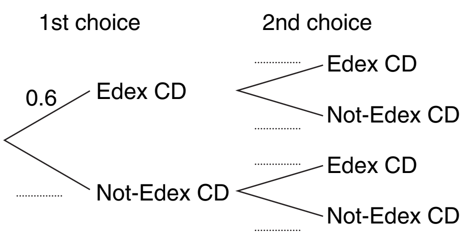
a Work out the probability that *both* Lauren *and* Yasmina will score a goal.

b Work out the probability that Lauren *will* score a goal *and* Yasmina *will not* score a goal.

(1385 June 1998)

11 Amy has 10 CDs in a CD holder.
Amy's favourite group is Edex.
She has 6 Edex CDs in the CD holder.
Amy takes one of these 10 CDs at random.
She writes down whether or not it is an Edex CD.
She puts the CD back in the holder.
Amy again takes one of these 10 CDs at random.

a Copy and complete the probability tree diagram.



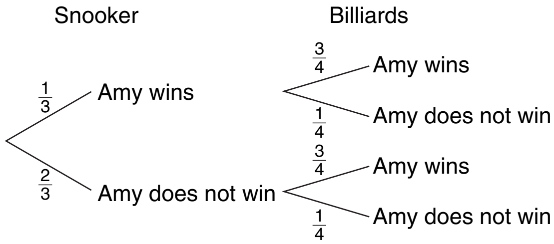
Amy had 30 CDs.
The mean playing time of these 30 CDs was 42 minutes.
Amy sold 5 of her CDs.
The mean playing time of the 25 CDs left was 42.8 minutes.

b Calculate the mean playing time of the 5 CDs that Amy sold.

(1387 June 2004)

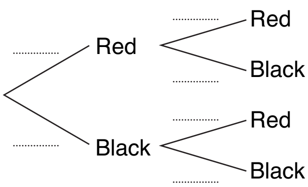
12 A bag contains 3 black beads, 5 red beads and 2 green beads.
Gianna takes a bead at random from the bag, records its colour and replaces it.
She does this two more times.
Work out the probability that of the three beads Gianna takes, exactly two are the same colour.
(1387 June 2003)

13 Amy is going to play one game of snooker and one game of billiards.
The probability that she will win the game of snooker is $\frac{1}{3}$
The probability that she will win the game of billiards is $\frac{3}{4}$
The probability tree diagram shows this information.



Amy played one game of snooker and one game of billiards on a number of Fridays.
She won at *both* snooker and billiards on 21 Fridays.
Work out an estimate for the number of Fridays on which Amy did not win either game.
(1388 June 2005)

14 A bag contains 10 coloured discs.
4 of the discs are red and 6 of the discs are black.
Asif is going to take two discs at random from the bag, *without* replacement.



a Copy and complete the tree diagram.

b Work out the probability that Asif will take two black discs.

c Work out the probability that Asif takes two discs of the same colour. (1385 May 2002)

15 Ali has twenty socks in a sock drawer.
10 of them are grey, 6 of them are black and 4 of them are red.
Ali takes two socks at random without replacement from the drawer.
Calculate the probability that he takes two socks that have the same colour. (1385 November 2001)

16 There are n beads in a bag.
6 of the beads are black and all the rest are white.
Heather picks one bead at random from the bag and does not replace it.
She picks a second bead at random from the bag.
The probability that she will pick 2 white beads is $\frac{1}{2}$
Show that $n^2 - 25n + 84 = 0$ (1385 June 2002)