Probability

Favourites to seize the Olympic flame

As the day for decision approaches it seems **unlikely** that London will win the battle to host the 2012 Olympic Games. The **probability** that Paris will win this race has always been high. It is felt that Madrid, Moscow and New York have little **chance** of success as the final presentations are made.

London team defies all the odds

London won with their bid to host the 2012 Summer Olympic Games. Yesterday's vote saw **likely** winners Paris stumble at the final hurdle. A spokesperson said 'Everyone thought that Paris was **certain** to win the vote, but I always felt that we had a greater than **even chance** of success.'

Probability is a measure of how likely it is that an event will happen.
An event could be **certain** to happen. For example, if it is Friday today, it is certain to be Saturday tomorrow.

An event could be **impossible** or have **no chance** of happening. For example, it is impossible for a baby to weigh 1000 kg.

An event which is as likely to happen as it is not to happen is said to have an **even** chance of happening. For example, there is an even chance of a spun coin coming down heads.
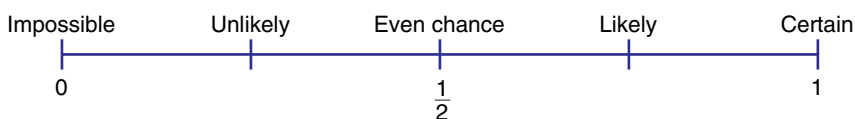
23.1 The probability scale

For an event, the probability of an outcome which is *certain* to happen is said to be **1**
The probability of an outcome which is *impossible* is said to be **0**
The probability of an outcome which has an *even chance* of happening is said to be **0.5** or $\frac{1}{2}$

This **probability scale** shows these.

Impossible	Unlikely	Even chance	Likely	Certain
0		$\frac{1}{2}$		1

An outcome which has less than an even chance of happening is said to be **unlikely**.
An outcome which has more than an even chance of happening is said to be **likely**.

Example 1

On the probability scale, mark with an arrow the probability that it will rain in Manchester during April.

Solution 1

Rain in Manchester

$$0 \qquad \frac{1}{2} \qquad 1$$

As it is likely that it will rain in Manchester during April, the probability is between $\frac{1}{2}$ and 1. So the arrow can be anywhere between $\frac{1}{2}$ and 1.

Exercise 23A

1 In each of the following choose the word from the list below that best describes the probability of each outcome happening.

| **Impossible   Unlikely   Even chance   Likely   Certain** |

a The sun will never shine again.

b It will snow in Scotland next year.

c The roll of a dice will show an odd number.

d The next baby to be born will weigh less than two tons.

e It will snow in London in July.

f The colour of a card picked from a pack of playing cards will be red.

g December will be the next month after November.

h Your maths teacher will be on television tonight.

2 Draw a probability scale like the one in Example 1. Mark with a ↓ the probability of each of the following outcomes. Label your arrows with the letters **a**–**d**.

a A spun coin will come down tails.

b This evening, in Wales, the sun will set in the West.

c Great Britain will win more gold medals than the USA in the next Summer Olympic Games.

d You will score more than 10% in your GCSE maths exam.

3 Write down one example of an event the outcome of which is:

a impossible   **b** unlikely   **c** even chance

d likely   **e** certain to happen.

23.2 Writing probabilities as numbers

The diagram shows a three-sided spinner.

The spinner can land on red or blue or yellow.
It is equally likely to land on each of the three
colours, so the spinner is said to be **fair**.
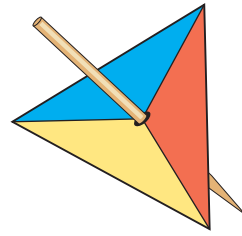
This spinner, which is fair, is spun once.
This is called a single **event**.
The colour that it lands on is called the **outcome**.
The outcome can be red or blue or yellow. There are three possible outcomes.

The probability that the spinner will land on blue $= \dfrac{1 \text{ successful outcome}}{3 \text{ possible outcomes}} = \dfrac{1}{3}$

Similarly, the probability that the spinner will land on red $= \dfrac{1}{3}$

and the probability that the spinner will land on yellow $= \dfrac{1}{3}$

When all the possible outcomes are equally likely to happen,

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

Probability can be written as a fraction or a decimal.

Example 2

A fair five-sided spinner is numbered 1 to 5
Jane spins the spinner once.

a Find the probability that the spinner will land on the
number 4

b Find the probability that the spinner will land on an even
number.
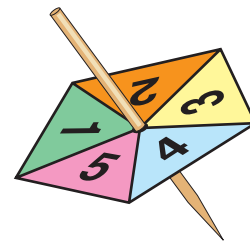
Solution 2

a The possible outcomes are 1, 2, 3, 4 and 5. There are **5 outcomes**.
The successful outcome is the number 4, just **1** outcome.
The probability that the spinner will land on $4 = \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}} = \dfrac{1}{5}$
or 0.2

b 2 and 4 are even numbers.
The number of successful outcomes is **2**
The total number of possible outcomes is **5**
So the probability that the spinner will land on an even number $= \dfrac{2}{5}$ or 0.4

Example 3

Six coloured counters are in a bag.
3 counters are red, 2 counters are green and 1 counter is blue.
One counter is chosen at random from the bag.
a Write down the colour of the counter which is
    i most likely to be chosen   **ii** least likely to be chosen.
b Find the probability that the counter chosen will be
    i red   **ii** green   **iii** blue.

Solution 3

In this question, at **random** means that each counter is equally likely to be chosen.
a   **i** Red is the most likely colour to be chosen since there are more red counters than green or blue.
    ii Blue is least likely to be chosen since there are fewer blue counters than red or green.
**b** There are 6 possible outcomes.

Outcome   1   2   3   4   5   6
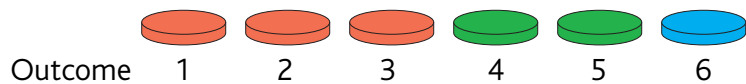
    i Number of successful outcomes = 3 (3 red counters).
    The probability that the counter will be red $= \frac{3}{6} = \frac{1}{2}$
    ii Number of successful outcomes = 2 (2 green counters).
    The probability that the counter will be green $= \frac{2}{6} = \frac{1}{3}$
    iii Number of successful outcomes = 1 (1 blue counter).
    The probability that the counter will be blue $= \frac{1}{6}$

Exercise 23B

1 Nicky spins the spinner. The spinner is fair.
Write down the probability that the spinner will land on a side coloured

    a blue   **b** red   **c** green.
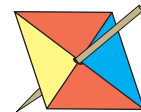
2 John spins the fair spinner.
Write down the probability that the spinner will land on

    a 2   **b** a number more than 5
    c an even number   **d** a number bigger than 10.

3

Jim has a bag of 6 counters. 3 of the counters are green, 1 is red, and the others are blue. Jim chooses a counter at random from the bag. Find the probability that Jim will choose

    a a red counter   **b** a blue counter   **c** a green counter.

4 Samantha Smith has eight cards which spell 'Samantha'.

She puts the cards in a bag and chooses one of the cards at random. Find the probability that she will choose a card showing

S A M A N T H A

   a a letter S

   b a letter A

   c a letter which is also in her surname 'SMITH'.

5 Ben has 15 ties in a drawer.
7 of the ties are plain, 3 of the ties are striped, and the rest are patterned.
Ben chooses a tie at random from the drawer. What is the probability that he chooses a tie which is

   a plain   **b** striped   **c** patterned?

6 Lisa has 5 cards numbered 1 to 5
Lisa shuffles the cards and chooses one of them at random.
What is the probability that the card will show

   1 2 3
   4 5

   a a 4   **b** an even number

   c an odd number   **d** a number bigger than 3

   e a number less than 10 **f** a 6?

7 Peter has a bag of 8 coins. In the bag he has one 10 p coin, five 20 p coins and the rest are 50 p coins. Peter chooses one coin at random. What is the probability that Peter will choose

   a a 10 p coin   **b** a 20 p coin

   c a 50 p coin   **d** a £1 coin

   e a coin worth more than 5 p?

8 Rob has a drawer of 20 socks.
4 of the socks are blue, 6 of the socks are brown, and the rest of the socks are black.
Rob chooses a sock at random from the drawer.
Find the probability that he chooses

   a a blue sock   **b** a brown sock

   c a black sock   **d** a white sock.

9 Verity has a box of pens.
Half of the pens are blue, 11 of the pens are green, 10 of the pens are red, and the remaining 4 pens are black.
Verity chooses a pen at random from the box.
Find the probability that she chooses

   a a blue pen   **b** a green pen

   c a red pen   **d** a black pen.

23.3 Two-way tables

This **two-way table** shows some information about the numbers of girls and boys in Year 10 and in Year 11 in a school.

	Year 10	Year 11	Total
Girls	80	90	170
Boys	75	105	180
Total	155	195	350

One of the ways shows information about the numbers of girls and boys.
The other way shows information about the number of students in Year 10 and the number of students in Year 11

Example 4

From the table, find the probability that a student chosen at random from all the students in Years 10 and 11 will be a Year 10 girl.

Give your answer in its simplest form.

Solution 4

The number of successful outcomes is 80 (because there are 80 Year 10 girls).
The number of possible outcomes is 350 (the total number of students in Years 10 and 11).
The probability that the student chosen will be a Year 10 girl $= \frac{80}{350} = \frac{8}{35}$

Sometimes a given two-way table is incomplete and the table has to be completed before probability questions can be answered.

Example 5

This two-way table gives information about the ages of students in a school.

	Under 12	12 to 15	Over 15	Total
Boys	96		35	
Girls		275		444
Total		500	100	

a Complete the two-way table.
b One of the students is to be picked at random.
   i Find the probability that the student picked will be 12 to 15
   ii Find the probability that the student picked will be a boy who is under 12

Give your answers in their simplest forms.

Solution 5

	Under 12	12 to 15	Over 15	Total
Boys	96	**225**	35	**356**
Girls	**104**	275	**65**	444
Total	**200**	500	100	**800**

a   The total number of girls over 15 is $100 - 35 =$ **65**
     The total number of girls under 12 is $444 - (275 + \mathbf{65}) =$ **104**
     The total number of students under 12 is $\mathbf{104} + 96 =$ **200**
     The total number of boys 12 to 15 is $500 - 275 =$ **225**
     The total number of students $= \mathbf{200} + 500 + 100 =$ **800**
     The total number of boys $= \mathbf{800} - 444 =$ **356**

Check: The total number of boys is $96 + \mathbf{225} + 35 =$ **356**

b   **i**   The probability that a student aged 12 to 15 will be picked $= \frac{500}{800} = \frac{5}{8}$
     ii   The probability that a boy under 12 will be picked $= \frac{96}{800} = \frac{3}{25}$

Example 6

Here is some information about the way that the boys and girls in class 10C travel to school.

Boys   **Girls**

Walk	Bus	Car	Bus		Bus	Bus	Car	Walk	Bus	Bus
Car	Bus	Bus	Bus		Bus	Car	Walk	Bus	Car	Car
Walk	Walk	Bus	Walk		Bus	Bus	Car	Walk	Bus	Car

a   Show this information in a suitable two-way table.

b   One of the students in 10C is to be chosen at random.
     Find the probability that this student will travel to school by bus.
     Give your answer in its simplest form.

c   One of the students in 10C who travels to school by car is to be chosen at random.
     Find the probability that this student will be a girl.
     Give your answer in its simplest form.

Solution 6

a

	Walk	Bus	Car	Total
Boys	4	6	2	12
Girls	3	9	6	18
Total	7	15	8	30

b   The probability that the student chosen will travel to school by bus $= \frac{15}{30} = \frac{1}{2}$

c   8 students travel to school by car. 6 of these 8 students are girls.
     The probability that the student chosen will be a girl $= \frac{6}{8} = \frac{3}{4}$

Exercise 23C

1 The two-way table gives some information about the weather on a number of days in the months of April, May and June.

a Write down the total number of rainy days.

b Write down the number of sunny days in June.

c Write down the number of days in May which were neither sunny nor rainy.

	Sunny	Rainy	Total
April	4	20	24
May	9	12	21
June	18	7	25
Total	31	39	70

2 The two-way table gives some information about the favourite fruit of a group of children.

a Copy and complete the two-way table.

b A child is chosen at random from the group. Find the probability that the child's favourite fruit will be oranges. Give your answer in its simplest form.

	Bananas	Oranges	Apples	Total
Boys	2	5		14
Girls		4	2	
Total	6			

c Find the probability that a child chosen at random from the group will be a girl whose favourite fruit is bananas. Give your answer in its simplest form.

3 The two-way table gives some information about the colours of some vehicles in a car park.

a Copy and complete the two-way table.

b Find the probability that a vehicle chosen at random will be a car.

	Silver	Black	White	Total
Cars		7		
Vans	3		8	
Total	15	12		40

Give your answer in its simplest form.

c Find the probability that a vehicle chosen at random will be a white van. Give your answer in its simplest form.

4 80 students were asked which football team they support. The two-way table shows some of their answers.

a Copy and complete the two-way table.

b Find the probability that a girl chosen at random will support Rovers.

	United	City	Rovers	Total
Boys			9	
Girls	10			32
Total		31	15	

Give your answer in its simplest form.

c Find the probability that a boy chosen at random will support United. Give your answer in its simplest form.

5   A group of 17 boys and 15 girls were asked at what time they went to lunch yesterday.

   12 boys went to lunch before 12 30
   13 students, of which 8 were girls, went to lunch from 12 30 to 13 00
   3 girls went to lunch after 13 00

|	Before 12 30	From 12 30 to 13 00	After 13 00	Total
Boys				
Girls				
Total				32

   a   Copy and complete the two-way table.

   b   A girl is chosen at random. Find the probability that the girl went to lunch before 12 30. Give your answer in its simplest form.

   c   A boy is chosen at random. Find the probability that he went to lunch after 13 00

23.4 Sample space diagrams

A **sample space diagram** shows all the possible outcomes of a single event or all the possible outcomes of more than one event.

For this three-sided spinner, the sample space when the spinner is spun once is

| 1 | 2 | 3 |
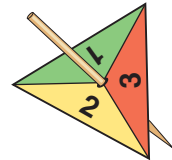
Example 7

The 3-sided spinner is spun and a coin is tossed at the same time.
Draw the sample space of all possible outcomes.

Solution 7
There are six possible outcomes.
The possible outcomes are 1 and a head, 1 and a tail, and so on.
The sample space is:

   (1, head)   (1, tail)   (2, head)   (2, tail)   (3, head)   (3, tail).

Example 8

Two fair dice are thrown.
a   Draw the sample space showing all the possible outcomes.
b   Find the probability that the numbers on the two dice will be
   i both the same   **ii** both even numbers   **iii** both less than 3

Solution 8

a (1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)
(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)
(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)
(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6)

A total of 36 possible outcomes.

b **i** (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) and (6, 6) are the successful outcomes with both numbers the same.

The probability that the numbers on both dice will be the same $= \frac{6}{36} = \frac{1}{6}$

ii (2, 2) (2, 4) (2, 6) (4, 2) (4, 4) (4, 6) (6, 2) (6, 4) (6, 6) are the successful outcomes with both numbers even.

The probability that the numbers on both dice will be even $= \frac{9}{36} = \frac{1}{4}$

iii (1, 1) (2, 1) (1, 2) (2, 2) are the successful outcomes with both numbers less than 3.

The probability that the numbers on both dice will be less than 3 is $\frac{4}{36} = \frac{1}{9}$

Exercise 23D

In each of these questions give all probabilities in their simplest forms.

1 Two coins are spun at the same time.
   a Draw the sample space showing all possible outcomes.
   b Find the probability that both coins will come down heads.
   c Find the probability that one coin will come down heads and the other coin will come down tails.

2 A bag contains 1 blue brick, 1 yellow brick, 1 green brick, and 1 red brick.
   A brick is chosen at random from the bag and its colour noted.
   The brick is then replaced in the bag.
   A brick is again chosen at random from the bag and its colour noted.
   a Draw a sample space to show all the possible outcomes of the colours of these two bricks.
   b Find the probability that
      i the two bricks will be the same colour
      ii one brick will be red and the other brick will be green.

3 Two fair dice are thrown.
   The sample space containing all the possible outcomes is shown in Example 8
   The numbers on the two dice are added together.
   a Find the probability that the sum of the numbers on the two dice will be
      i greater than 10   **ii** less than 6   **iii** a square number.
   b **i** What sum of the numbers on the two dice is most likely to occur?
      ii Find the probability of this sum occurring.

4 Daniel has four cards – the ace of hearts, the ace of diamonds, the ace of spades and the ace of clubs.
Daniel also has a fair dice.
He rolls the dice and chooses a card at random.
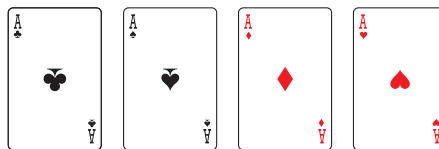
  a Draw the sample space showing all possible outcomes. One possible outcome, ace of diamonds and 4 has been done for you.

      (D, 4)

  b Find the probability that a red ace will be chosen.

  c Find the probability that he will choose the ace of spades and roll an even number on the dice.

5 Three normal coins are spun.

  a Draw a sample space showing all eight possible outcomes.

  b Find the probability that the three coins will land on the same side.

  c Find the probability that the coins will show two heads and a tail.

  d Write down the number of possible outcomes when
    i four coins are spun   **ii** five coins are spun.

23.5 Mutually exclusive outcomes and the probability that the outcome of an event will not happen

Five coloured counters are in a bag.
3 counters are red and 2 counters are green.
One counter is chosen at random from the bag.

The probability of choosing a red counter $= \frac{3}{5}$

The probability of choosing a green counter $= \frac{2}{5}$

Mutually exclusive outcomes are outcomes which cannot happen at the same time.

When one counter is chosen at random from the bag, the outcome 'red' cannot happen at the same time as the outcome 'green'. The two outcomes are mutually exclusive.

Probability of red + probability of green $= \frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$

The sum of the probabilities of all the possible mutually exclusive outcomes of an event is 1

There are 5 possible outcomes, 2 of which are green.

The probability that the counter will be green is $\frac{2}{5}$

Out of the 5 possible outcomes, $5 - 2 = 3$ outcomes are NOT green.

The probability that the counter will NOT be green is $1 - \frac{2}{5} = \frac{3}{5}$

If the probability of something happening is p, then the probability of it NOT happening is $1 - p$.

Example 9

David buys one newspaper each day. He buys the Times or the Telegraph or the Independent. The probability that he buys the Times is 0.6. The probability that he buys the Telegraph is 0.25. Work out the probability that David will buy the Independent.

Solution 9

P(Times) = 0.6   P(Telegraph) = 0.25   P(Independent) = ?

> P(Times) means the probability that David buys the Times.

As David only buys one newspaper each day, the three outcomes are mutually exclusive.
P(Independent) + 0.6 + 0.25 = 1
P(Independent) + 0.85 = 1
P(Independent) = 1 − 0.85

The probability that David will buy the Independent = 0.15

Example 10

The probability that Julie will pass her driving test next week is 0.6
Work out the probability that Julie will NOT pass her driving test next week.

Solution 10

The probability that Julie will NOT pass = 1 − 0.6 = 0.4

Exercise 23E

1   Nosheen travels from home to school. She travels by bus or by car or by tram. The probability that she travels by bus is 0.4. The probability that she travels by car is 0.5 Work out the probability that she travels by tram.

2   Roger's train can be on time or late or early. The probability that his train will be on time is 0.15. The probability that his train will be early is 0.6. Work out the probability that Roger's train will be late.

3   The probability that Lisa will pass her maths exam is 0.8
Work out the probability that Lisa will not pass her maths exam.

4   A company makes batteries. A battery is chosen at random. The probability that the battery will **not** be faulty is 0.97. Work out the probability that the battery will be faulty.

5   Kevin has a bag of sweets. He takes a sweet at random from the bag. The probability that the sweet will be orange is 0.64. Work out the probability that the sweet will not be orange.

6   Four athletes, Aaron, Ben, Carl and Des, take part in a race. The table shows the probabilities that Aaron or Ben or Carl will win the race.

Aaron	Ben	Carl	Des
0.2	0.14	0.3	x

   a   Work out the probability that Aaron will not win the race.
   b   Work out the probability that Ben will not win the race.
   c   Work out the probability, x, that Des will win the race.

7　A roundabout has four roads leading from it. Michael is driving round the roundabout. The roads lead to Liverpool or Trafford Park or Eccles or Bolton. The table shows the probabilities that Michael will take the road to Liverpool or Trafford Park or Bolton.

Liverpool	Trafford Park	Eccles	Bolton
0.49	0.18	x	0.23

    a　Work out the probability that Michael will not take the road to Liverpool.

    b　Work out the value of x.

8　Sam has red, white, yellow and green coloured T-shirts only. She chooses a T-shirt at random. The probabilities that Sam will choose a red T-shirt or a white T-shirt are given in the table. Sam is twice as likely to choose a green T-shirt as she is to choose a yellow T-shirt.

Work out the value of x.

Red	White	Yellow	Green
0.5	0.14	x	$2x$

23.6 Estimating probability from relative frequency

The diagram shows two three-sided spinners.

One spinner is fair and one is biased.

A spinner is **biased** if it is not equally likely to land on each of the numbers.

This can be tested by experiment.

If a spinner is spun 300 times, it is fair if it lands on each of the three numbers approximately 100 times.

John spins one spinner 300 times and Mary spins the other spinner 300 times.
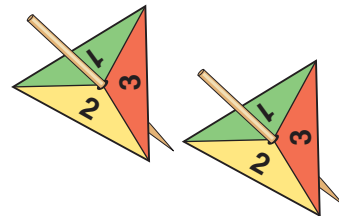
John's spinner			Mary's spinner		
300 spins			300 spins		
1	2	3	**1**	**2**	**3**
97	104	99	147	96	57

John's spinner is fair because it lands on each of the three numbers approximately the same number of times.

Mary's spinner is biased because it is more likely to land on the number **1** (147 times out of 300). It is least likely to land on the number **3** (only 57 times out of 300).

To estimate the probability that Mary's spinner will land on a particular number, the **relative frequency** of each number is found using

$$\text{relative frequency} = \frac{\text{number of times the spinner lands on the number}}{\text{total number of spins}}$$

The relative frequency that Mary's spinner will land on the number **1** $= \frac{147}{300} = 0.49$

The relative frequency that Mary's spinner will land on the number **2** $= \frac{96}{300} = 0.32$

The relative frequency that Mary's spinner will land on the number **3** $= \frac{57}{300} = 0.19$

An **estimate** of the probability that the spinner will land on the number **1** is 0.49
An estimate of the probability that the spinner will land on the number **2** is 0.32
An estimate of the probability that the spinner will land on the number **3** is 0.19

Example 11

In a statistical experiment, Brendan throws a dice 600 times. The table shows the results.

Number on dice	1	2	3	4	5	6
Frequency	48	120	180	96	54	102

Brendan throws the dice again.

a Estimate the probability that he will throw a 2

b Estimate the probability that he will throw an even number.

Zoe now throws the same dice 200 times.

c Estimate the number of times Zoe will throw a 6

Solution 11

a Relative frequency of a 2 is $\frac{120}{600}$

Estimated probability of a 2 is $\frac{120}{600} = 0.2$

b The number of times he throws an even number $= 120 + 96 + 102 = 318$

Relative frequency of an even number $= \frac{318}{600}$

Estimated probability of an even number $\frac{318}{600} = 0.53$

c Relative frequency of a 6 is $\frac{102}{600} = 0.17$

$$\text{Relative frequency} = \frac{\text{number of times Zoe throws a 6}}{\text{total number of throws}}$$

So an estimate for the number of times Zoe throws a 6 is $0.17 \times 200 = 34$

Exercise 23F

1 A coin is biased. The coin is tossed 200 times.
It lands on heads 140 times and it lands on tails 60 times.

   a Write down the relative frequency of the coin landing on tails.

   b The coin is to be tossed again. Estimate the probability that the coin will land on

      i tails   **ii** heads.

2 A bag contains a red counter, a blue counter, a white counter and a green counter. Asif chooses a counter at random. He does this 400 times. The table shows the number of times each of the coloured counters are chosen.

Red	Blue	White	Green
81	110	136	73

     a Write down the relative frequency of Asif choosing the red counter.

     b Write down the relative frequency of Asif choosing the white counter.

     Asif chooses a counter one more time. Estimate the probability that this counter will be

     c   **i** blue   **ii** green.

3 Ian throws a dice 400 times. He scores three 100 times. Is the dice fair? Explain your answer.

4 Tyler carries out a survey about the words in a newspaper. He chooses an article at random. He counts the number of letters in each of the first 150 words of the article. The table shows Tyler's results.

Number of letters in a word	1	2	3	4	5	6	7	8	9	10
Frequency	7	14	42	31	21	13	10	6	4	2

     A word is chosen at random from the 150 words.

     a Write down the most likely number of letters in the word.

     b Estimate the probability that the word will have

       i 1 letter   **ii** 7 letters   **iii** more than 5 letters.

     c The whole article has 1000 words. Estimate the total number of 3-letter words in this article.

5 A bag contains ten coloured bricks. Each brick is white or red or blue. Alan chooses a brick at random from the ten bricks in the bag. He does this 500 times. The table shows the numbers of each coloured brick chosen.

White	Red	Blue
290	50	160

     a Estimate the number of red bricks in the bag.

     b Estimate the number of white bricks in the bag.

6 The probability that someone will pass their driving test at the first attempt is 0.45 On a particular day, 1000 people take the test for the first time. Work out an estimate for the number of these 1000 people who pass.

7 Gwen has a biased coin. When she spins the coin, the probability that it will come down tails is $\frac{3}{5}$. Work out an estimate for the number of tails she gets when she spins her coin 400 times.

8 The probability that a biased dice will land on a one is 0.09. Andy is going to roll the dice 300 times. Work out an estimate for the number of times the dice will land on a one.

Chapter summary

You should know that:

★ **probability** is a measure of how likely it is that the outcome of an event will happen.

★ probabilities are written as fractions or decimals between 0 and 1

★ an outcome which is **impossible** has a probability of 0; an outcome which is **certain** to happen has a probability of 1

★ probabilities can be shown on a **probability scale**

★ in an event, outcomes which are equally likely have equal probabilities

★ when all outcomes of an event are equally likely to happen

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

★ two-way tables can be used to find probabilities; you should know how to complete a two-way table

★ outcomes can be listed in an ordered way using **sample space diagrams**

★ **mutually exclusive outcomes** are outcomes which cannot happen at the same time

★ **the sum of the probabilities of all the possible mutually exclusive outcomes is 1**

★ if the probability of something happening is p, then the probability of it NOT happening is $1 - p$

★ from a statistical experiment, for each outcome

$$\text{relative frequency} = \frac{\text{number of times the outcome happens}}{\text{total number of trials of the event}}$$

★ relative frequencies give good estimates of probabilities when the number of trials is large

★ if the probability that an experiment will be successful is p and the experiment is carried out a number of times, then an estimate for the number of successful experiments is $p \times$ number of experiments.
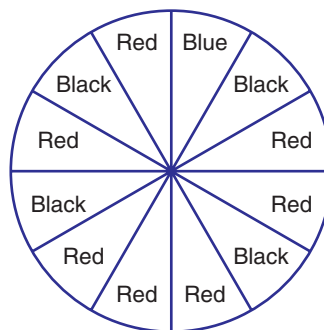
Chapter 23 review questions

1   On a probability line, mark the following probabilities.
   a   It will rain sometime next year in England. Use the letter R.
   b   The Sun will collide with the Earth tomorrow. Use the letter S.
   c   A fair coin when tossed will come down tails. Use the letter T.

2 Here is a spinner.
The spinner is spun.

a **i** Which colour is least likely?
ii Give a reason for your answer.

b On a probability line, mark with an X the probability that the colour will be Red.

c Write down the probability that the colour will be Blue.

(1386 June 1999)

3 Some bulbs were planted in October.
The ticks in the table show the months in which each type of bulb grows into flowers.

| | | Month | | | | | |
		Jan	**Feb**	**March**	**April**	**May**	**June**
	Allium					✓	✓
Type of bulb	Crocus	✓	✓				
	Daffodil		✓	✓	✓		
	Iris	✓	✓				
	Tulip				✓	✓	

a In which months do tulips flower?
b Which type of bulb flowers in March?
c In which month do most types of bulb flower?
d Which type of bulb flowers in the same months as the iris?

Ben puts one of each type of these bulbs in a bag.
He takes a bulb from the bag without looking.

e **i** Write down the probability that he will take a crocus bulb.
ii On a probability scale, mark with a cross (✗) the probability that he will take a bulb which flowers in February.

(1387 June 2005)

4 Richard has a box of toy cars. Each car is red or blue or white.
3 of the cars are red. 4 of the cars are blue. 2 of the cars are white.
Richard chooses one car at random from the box.
Write down the probability that Richard will choose a blue car. (1388 Jan 2004)

5 Shreena has a bag of 20 sweets. 10 of the sweets are red. 3 of the sweets are black.
The rest of the sweets are white.
Shreena chooses one sweet at random.
What is the probability that Shreena will choose a

a red sweet   **b** white sweet?   (1385 June 1999)

6 30 students were asked if they liked coffee.
20 of the students were girls.
6 boys liked coffee.
12 girls did **not** like coffee.

Use this information to copy and complete the two-way table.

	Boys	Girls	Total
Liked coffee			
Did not like coffee			
Total			

(1388 Jan 2004)

7 80 students each study one of three languages.
The two-way table shows some information about these students.

	French	German	Spanish	Total
Female	15			39
Male		17		41
Total	31	28		80

a Copy and complete the two-way table.

One of these students is to be picked at random.

b Write down the probability that the student picked studies French.

(1387 June 2005)

8 80 students were each asked to name their favourite swimming stroke.
The two-way table shows some information about these students' answers.

	Breaststroke	Front crawl	Backstroke	Total
Boy		28	7	
Girl	23			36
Total			12	80

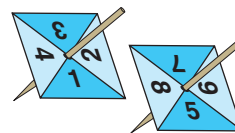**a** Copy and complete the two-way table.

One of these 80 students is picked at random.

b Write down the probability that this student's favourite stroke is backstroke.

9 Zen has two fair spinners.

One of the spinners is numbered 1, 2, 3, 4
The other spinner is numbered 5, 6, 7, 8

Zen is going to spin the two spinners.
He multiplies the numbers on the two spinners to get the score.

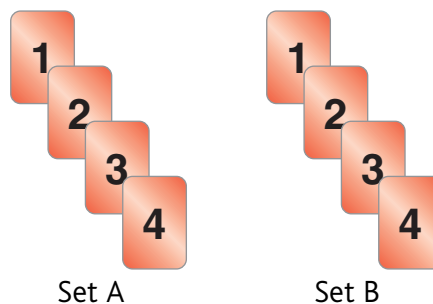**a** Copy and complete the table to show all the possible scores.

×	1	2	3	4
5	5	10	15	20
6	6			24
7	7			28
8	8	16	24	32

b What is the probability of getting a score of 28?

c What is the probability of getting a score of more than 25?   (1386 Nov 2001)

10 Here are two sets of cards. Each card has a
number on it as shown.
A card is selected at random from Set A and a
card is selected at random from Set B.

The difference between the number on the card
selected from Set A and the number on the card
selected from Set B is worked out.

Set A   Set B

a Copy and complete the table to show all the possible differences.

Set A

Set B		1	2	3	4
	1	0		2	
	2		0		
	3		1		
	4				

b Find the probability that the difference will be zero.

c Find the probability that the difference will **not** be 2

11 A company makes hearing aids. A hearing aid is chosen at random.
The probability that it has a fault is 0.09
Work out the probability that a hearing aid, chosen at random,
will **not** have a fault.

(1388 March 2004)

12 There are 20 coins in a bag.
7 of the coins are pound coins.
Gordon is going to take a coin at random from the bag.

   a Write down the probability that he will take a pound coin.

   b Find the probability that he will take a coin which is **not** a pound coin.

13 Asif's bus could be on time or late or early.
The probability that his bus will be on time is 0.9
The probability that his bus will be late is 0.03
Work out the probability that Asif's bus will be early.

(1385 Nov 1998)

14 Mr Brown chooses one book from the library each week.
He chooses a crime novel or a horror story or a non-fiction book.
The probability that he chooses a horror story is 0.4
The probability that he chooses a non-fiction book is 0.15
Work out the probability that Mr Brown chooses a crime novel.

(1387 June 2005)

15 Martin bought a packet of mixed flower seeds.
The seeds produce flowers that are red or blue or white or yellow.
The probability of a flower seed producing a flower of a particular colour is:

Colour	Red	Blue	White	Yellow
Probability	0.6	0.15		0.15

   a Write down the most common colour of a flower.

   Martin chooses a flower seed at random from the packet.

   b **i** Work out the probability that the flower produced will be white.
       ii Write down the probability that the flower produced will be orange.

(1385 June 1998)

16 Here is a 4-sided spinner.
The sides are labelled red, blue, yellow and green.
The spinner is biased.
The probability that the spinner will land on each of the
colours red, blue, and green is shown in the table.
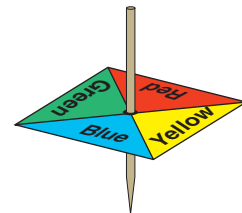
Colour	Red	Blue	Yellow	Green
Probability	0.45	0.3	x	0.1

Jo spins the spinner once.

   a Work out the probability, x, that the spinner will land on yellow.

   Daniel spins the spinner 100 times.

   b Work out an estimate for the number of times the spinner will land on blue.

17 Here is a 4-sided spinner.
The sides of the spinner are labelled 1, 2, 3 and 4
The spinner is biased.
The probability that the spinner will land on each of the
numbers 2 and 3 is given in the table.
The probability that the spinner will land on 1 is equal to the probability
that it will land on 4

Number	1	2	3	4
Probability	x	0.3	0.2	x

a Work out the value of x.

Sarah is going to spin the spinner 200 times.

b Work out an estimate for the number of times the dice will land on 2

(1387 June 2005)

18 The probability that a biased dice will land on a four is 0.2
Pam is going to roll the dice 200 times.
Work out an estimate for the number of times the dice will land on 4

(1387 June 2004)

19 Meg has a biased coin.
When she spins the coin, the probability that it will come down heads is 0.4
Meg is going to spin the coin 350 times.
Work out an estimate for the number of times it will come down heads.

20 Julie does a statistical experiment. She throws a dice 600 times.
She scores six 200 times.
Is the dice fair? Explain your answer.

(1387 June 2003)